MiniTest 4 Review Key

Dr. Graham-Squire, Spring 2013

1. Find the Maclaurin series for the functions $f(x) = e^{x^2}$ and $f(x) = x \sin x$.

   Ans: $e^{x^2} = \displaystyle\sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$, radius of convergence of infinity.

   Ans: $x \sin x = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+2}}{(2n+1)!}$, radius of convergence of infinity.

2. Let $f(x) = x^{2/3}$.

   (a) Approximate f by a Taylor polynomial with degree 3 at $a = 1$.

   Ans: $T_3(x) = 1 + \dfrac{2}{3}(x-1) - \dfrac{1}{9}(x-1)^2 + \dfrac{4}{81}(x-1)^3$

   (b) Use Taylor's inequality to estimate the accuracy of the approximation $f(x) = T_3(x)$ when x is in the interval $[0.8, 1.2]$.

   Ans: Using $M = 1.403$, find the remainder to be less than 0.0000935

   (c) Check the accuracy of your result from (b) by graphing $|R_3(x)|$.

   Ans: If you graph $|x^{2/3} - (1 + \frac{2}{3}(x-1) - \frac{1}{9}(x-1)^2 + \frac{4}{81}(x-1)^3)|$ you should find that the largest error occurs at the endpoints, and it looks to be somewhat in excess of 0.00005. This seems about right, since Taylor's inequality always gives you an overestimate of the error.

3. Determine if the following points are collinear, and explain your answer:

   $$P = (2, -1, 5) \qquad Q = (8, 3, 13) \qquad R = (-7, -7, -7).$$

   Ans: Yes, they are collinear! The change from P to Q is 6 up, 4 up, 8 up, and the change from P to R is 9 down, 6 down, 12 down. Since 6:4:8 and -9:-6:-12 are in the same ratio, the points all lie on the same line.

4. Sketch the surface given by the equation $4x^2 - 9y^2 = -4z^2$.

   Ans: This will be an elliptic cone with y as the rotational axis. You can either think of it as a quadric surface or a surface of revolution when you graph it.

5. (a) The point $(2, 2\pi/3, -2)$ is in cylindrical coordinates. Convert it to spherical coordinates.

   Ans: $(2\sqrt{2}, 2\pi/3, 3\pi/4)$

   (b) Find an equation in rectangular coordinates for the equation $z = r^2 \cos^2 \theta$ given in cylindrical coordinates. Sketch and/or describe the graph.

   Ans: $z = x^2$. The graph will be a cylindrical surface with a parabola for a generating curve and rulings parallel to the y-axis.